

TM acceptor
 $\hookrightarrow w \in L$
 $w \notin L$

TM transducer
 output

TRANSDUCER

Eq: TM to find 1's complement of a binary no.

Input: 0011 \dots

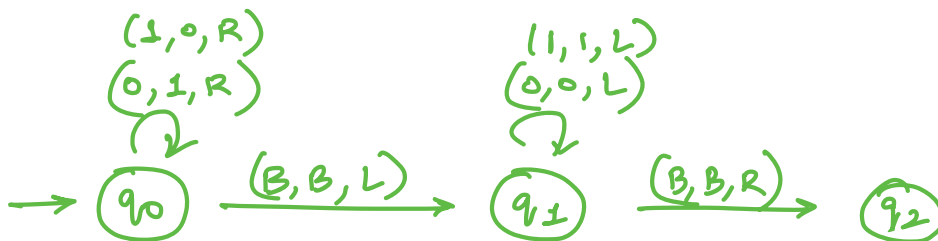
B	0	0	1	1	B
---	---	---	---	---	---

 \dots

Output: 1100 \dots

B	1	1	0	0	B
---	---	---	---	---	---

 \dots



Eq: TM for computing 2's complement

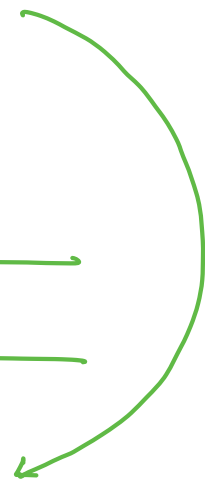
Original: 011101000

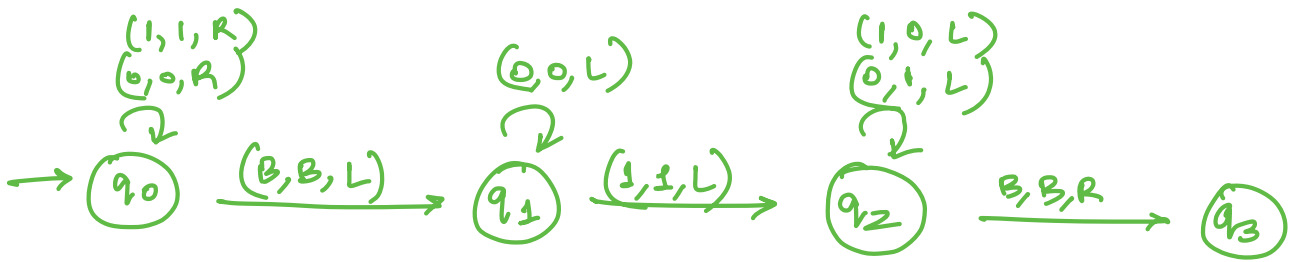
1's Comp: 100010111

+
 +

 100011000

100011000





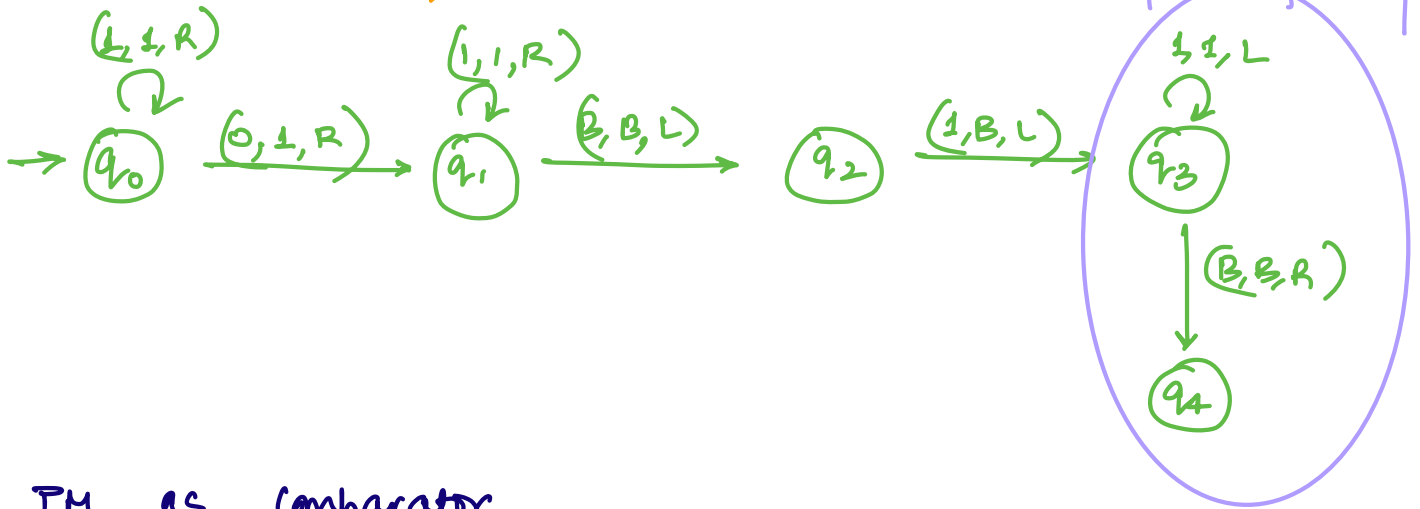
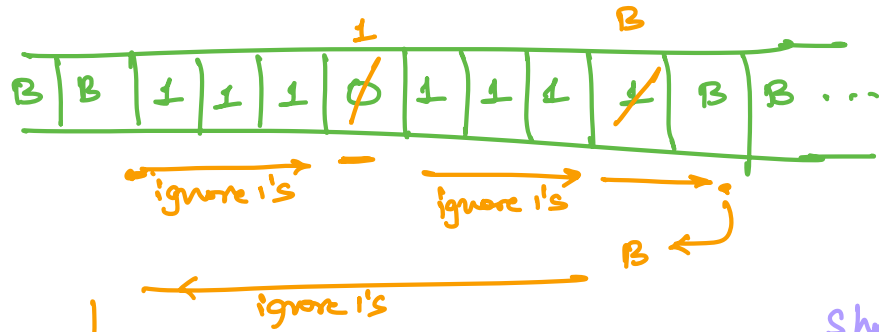
Eg: TM as adder

3 → 111

|x| = no.

4 → 1111

2 → 11



Eg: TM as Comparator

a b

a = b

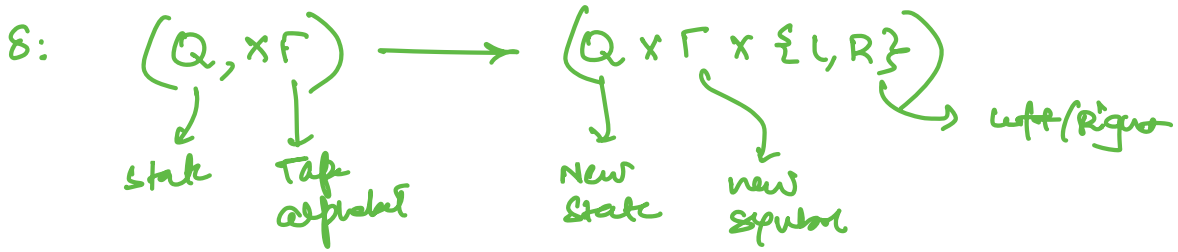
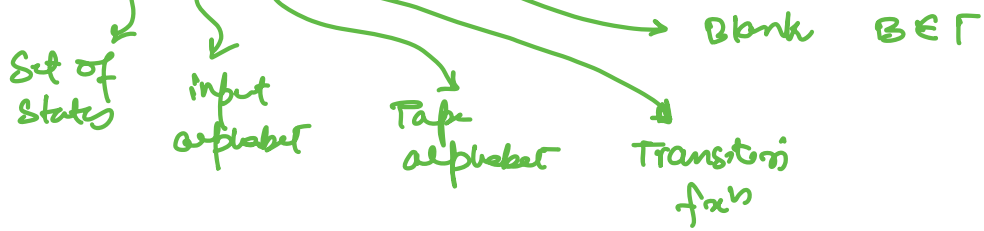
a > b

a < b

$$\frac{1110111}{3} = \frac{1111111}{3} \quad a = b$$

$$\frac{11110111}{4} > \frac{1111111}{3} \quad a > b$$

$$\frac{111011111}{3} < \frac{111111111}{3} \quad a < b$$



Properties:

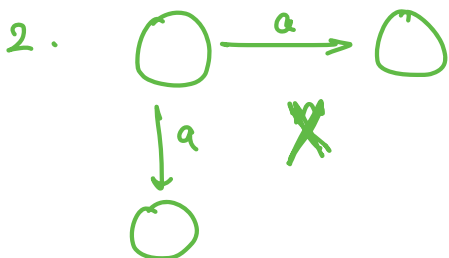
1. Tape is unbounded, you can take any no. of left & right steps.
2. TM is deterministic

fully Deterministic

Partially Deterministic



from every state for every input alphabet there will be a transition

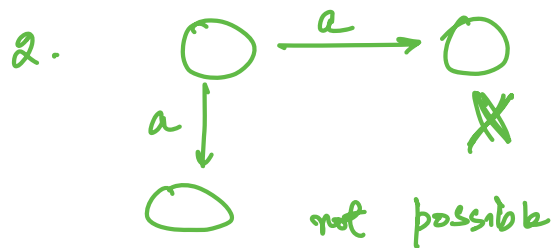


$(Q, \Sigma) \rightarrow$ exactly one transition

Eg: DFA

1. for every state for every alphabet you don't have to show transitions

Eg: TM, PDA



Non Deterministic Eg: NFA, ENFA

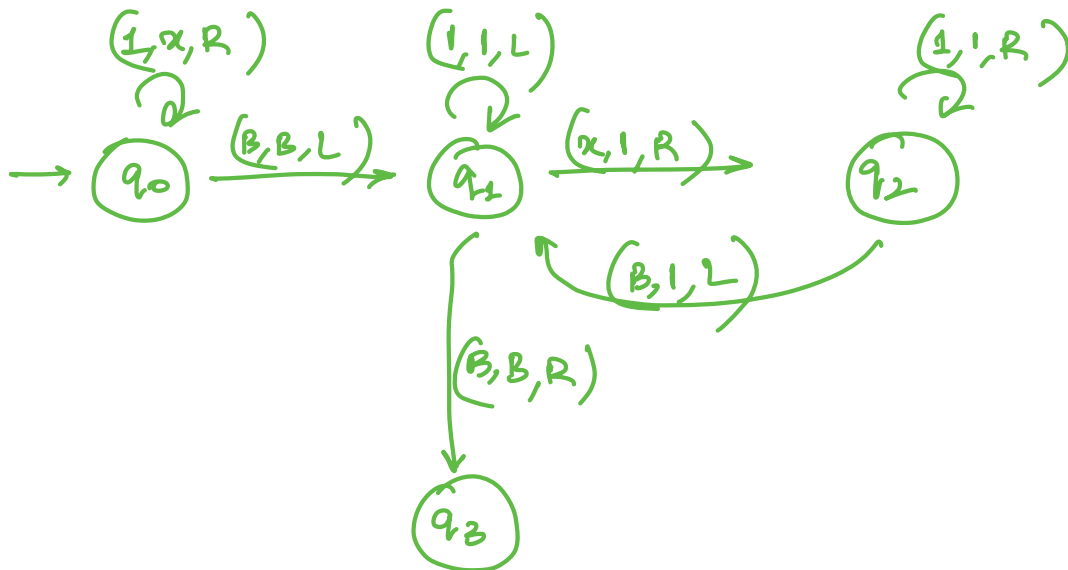
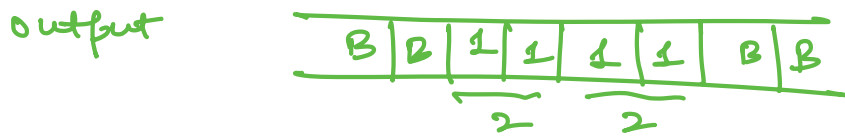
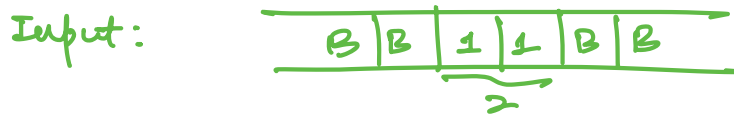
FA + 1 Stack = PDA

PDA + 1 Stack = TM

FA + 2 Stack = TM

FA + Queue = TM

Eg: TM as a copier $\xrightarrow{\quad} a \xrightarrow{\quad} 2a$



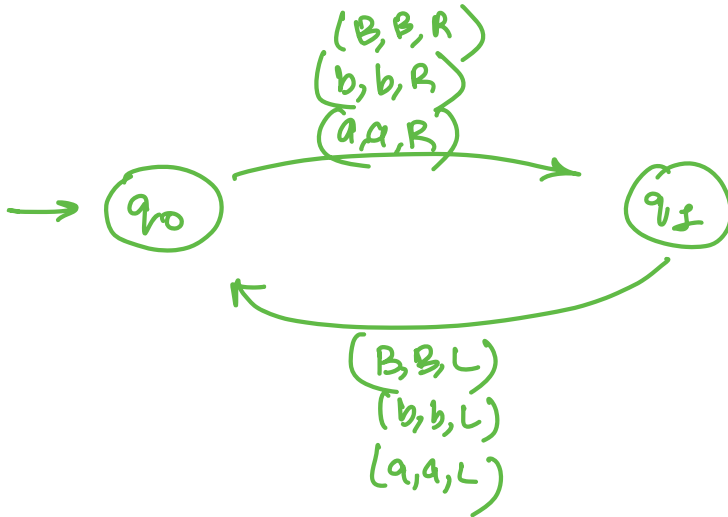
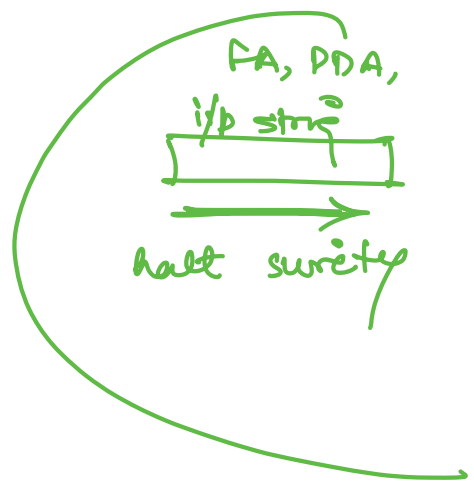
$3+3 = 2+3$

$2+3$
 $2+4$

TM which can do multiplication of

TM is mathematically complete

Non Halting TM



halting problem
becz of i/p string
you can move
in both dir's



$a^n b^n$ TM m/c

I/p string: aabb

Input string is in the language

TM will halt
& it will halt at final state

I/p string: aab

Input string is not in the language

TM halt at
a non final state

TM never halts. ??

↓ Problems

Problem with non halting
TM is you don't know
how long to wait
↓
non halting TM Problem

Case 1:
TM is doing
some
computation

Case 2:
TM is stuck in
 ∞ loop

U feel: ∞ loop
stuck
Stop the TM

U feel:
computation

Turing Thesis

Turing is a scientist, hypothesis 1930

The
imitation
game

Any computation that can be
carried out by mechanical means
can be performed by TM.
→ TM is as powerful as a computer

Prove?

$P_1 \rightarrow TM_1$
 $P_2 \rightarrow TM_2$
 P_3

$P_\infty \rightarrow TM_\infty$

Issue: ∞ Problems

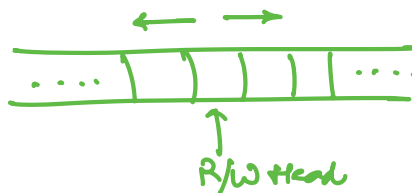
Alternatively, you can come up with a problem that is
not solvable by your TM.

→ nobody was able to come up with a problem
which is not solvable by TM.

- TM & Computers are equally powerful.

Modifications / Variants of standard TM:

Standard TM:



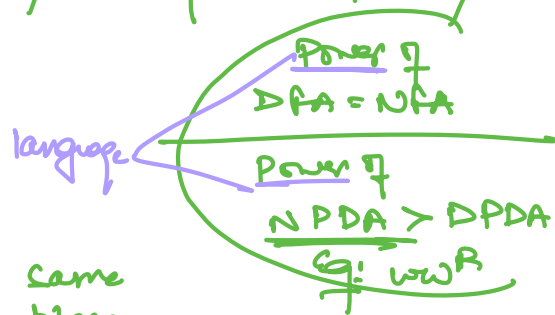
Power of TM: language accepted by TM.

→ not the time complexity or space complexity

① TM with stay option

Standard TM: left, right

Modified TM: will remain at the same place.



$$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, S\}$$

Power of this TM = Power of Standard TM

② TM with semi infinite tape

Standard TM:

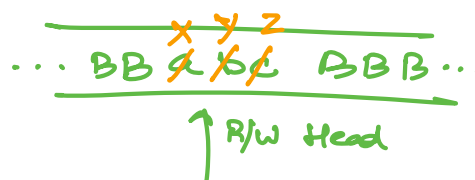


Semi-infinite TM:

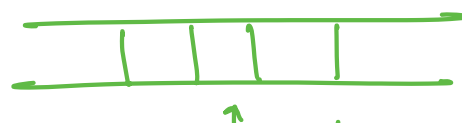


③ Offline TM

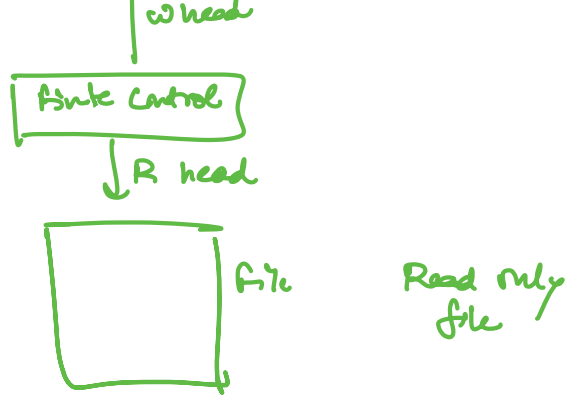
Standard TM:



Finik Control



writing Tape

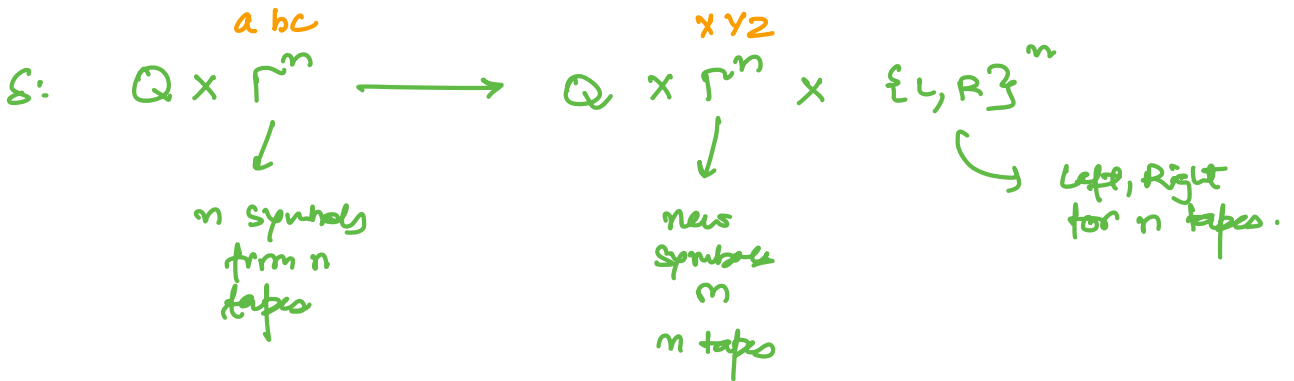
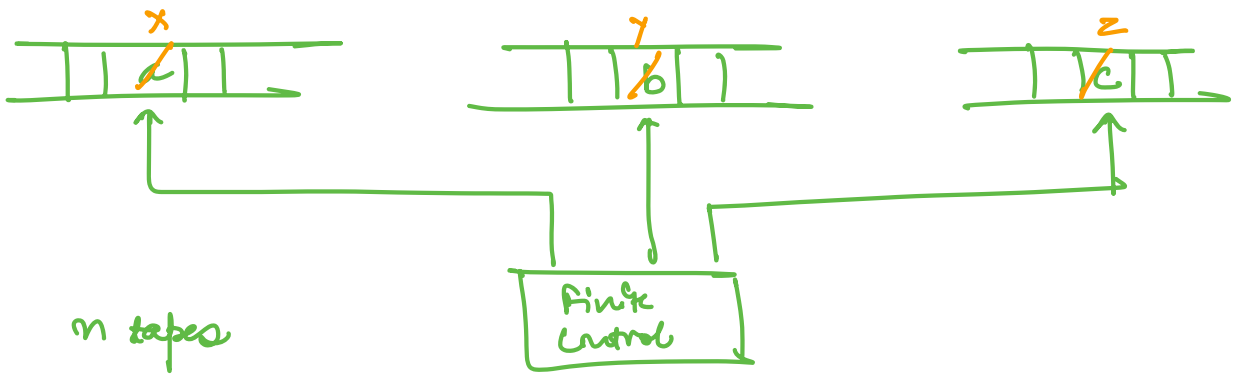


④ Multitape TM

Standard TM:



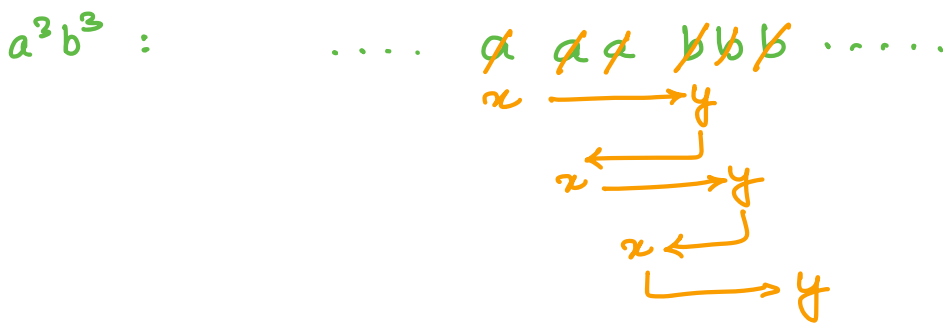
Multitape TM:



Power of Standard TM = Power of multitape TM

Benefit of Multitape TM is time is reduced.

Eg: $a^n b^n$



Standard TM

You want to match n pairs
 for every pair u have to move n steps.
 for n pairs u have to move n^2 steps.
 $TC = O(n^2)$

Multitape TM:

$a^n b^n$

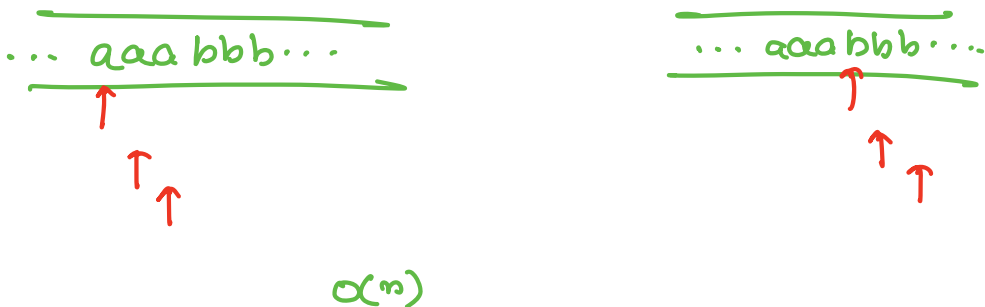
① copy the entire input to other tape



② set the read/write head



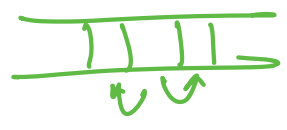
③ scan symbols one by one in both the tapes



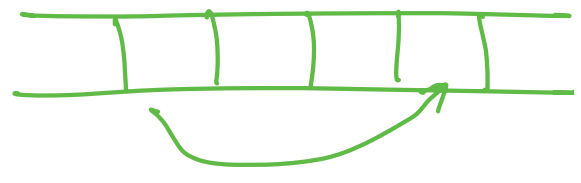
Total time: $2n = O(n)$

⑤ jumping TM

Standard TM:



jumping TM:

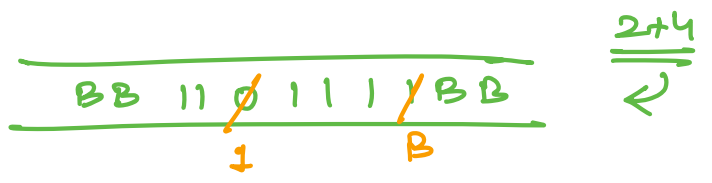


$$\delta: Q \times \Gamma = Q \times \Gamma \times \{L, R\} \times \{n\}$$

↓
steps in jump

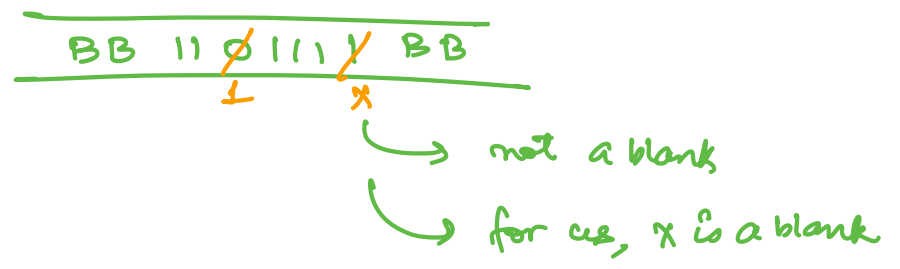
⑥ Non Erasing TM

Standard TM:



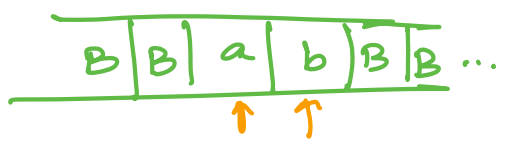
Input Symbol \leftrightarrow blank

Non Erasing TM: Remove the option of changing input to blank.



⑦ Always writing TM

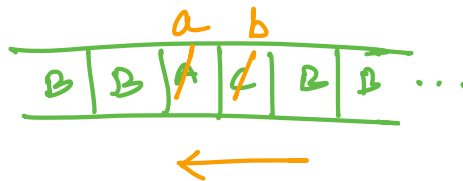
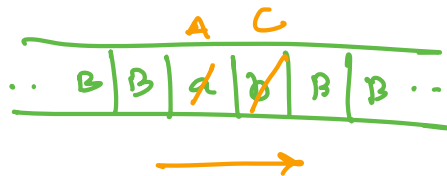
Standard TM:



You may not change the tape alphabet

Always writing TM:

Definitely change the input alphabet

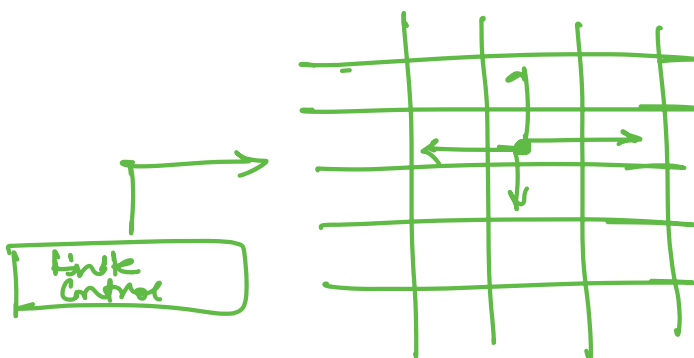


⑧ Multi dimensional TM

Standard TM:



Multi dimension:

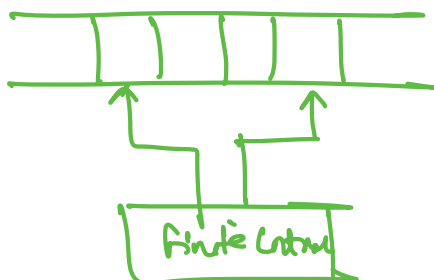


Power is same

$$\delta: Q \times \Gamma \rightarrow Q \times \Gamma \times \{L, R, U, D\}_2^2$$

⑨ Multihead TM

Single tape, read the content from multiple places at same time



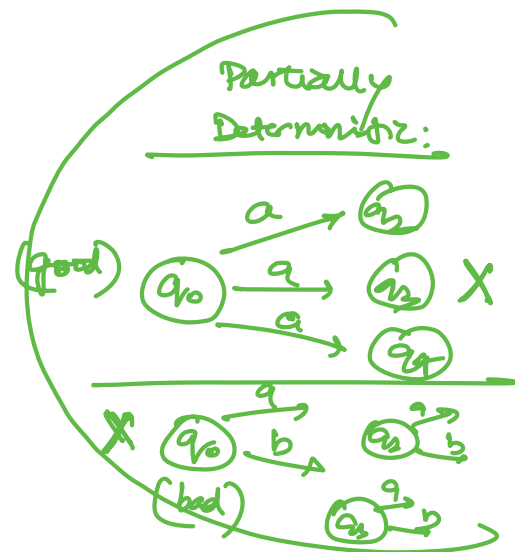
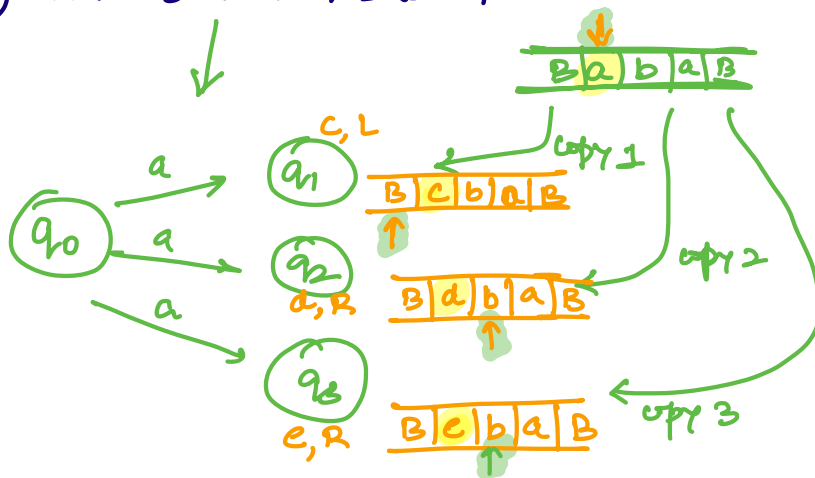
⑩ Automata with queue

TM: Automata + Queue

⑪ Any TM can be minimized to a TM with only 3 states

⑫ Any Standard TM can be converted to a multitape TM with stay option and atleast 2 states.

⑬ Non deterministic TM



on looking at one state and one symbol we can make multiple

copies & can simultaneously go to many states and can change the tape symbol

$$\delta: Q \times \Gamma \rightarrow 2^{Q \times \Gamma \times \{L, R\}}$$

Non Deterministic TM & Deterministic TM have equal power.

$$DTM \cong NTM$$

